# Effiziente Algorithmen

Summer term 2012

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# Assignment 7

Take home: 05/**29**/2012
Submit: 06/04/2012

*Note*: **It is not particularly smart to post exercises on various discussion boards and copy solutions that are made intentionally false (and that are even marked as being so!).**

*Note*: It is understood that all of your statements have to be proven correct.

*Note*: Solutions may be submitted by email. Solutions submitted after the lecture will not be graded.

*Note*: All points of this assignment are bonus points.

## Exercise 7.1. (2+3+3) *Streaming vs. random access*

Consider an array $A$ of $n$ cells, each containing a number of $\{1, \ldots, n-1\}$. Observe that there is at least one duplicate number, i.e., a number that appears at least twice. Our task is to output *some* duplicate number. When streaming we may pass over $A$ more than once. The inspection of a cell generates cost 1. The cost of a run of an algorithm is the sum of all individual costs. We are free to store one $\log_2 n$ bit number.

- State a streaming algorithm that uses additional memory $O(1)$ with costs $O(n^2)$.
- State a streaming algorithm that uses additional memory $O(\log_2 n)$ with costs $O(n \log_2 n)$.
- State a random access algorithm that uses additional memory $O(\log_2 n)$ with costs $O(n)$.

  *Hint*: Let $S$ be finite, $f : S \to S$ and $x_0 \in S$ be given. Let $f^m(x_0)$ be the first reoccuring item in an iterated application of $f$ and let $l$ be the 'loop length'. The cycle detection problem is to find $f^m(x_0)$ and $l$. You may utilize that the cycle detection problem can be solved in time $O(|S|)$ storing only two items of $S$.

## Exercise 7.2. (8) *Byzantine Agreement*

We consider the Byzantine Agreement algorithm 2.24 of the lecture notes.

Show how to adjust the thresholds LOW, HIGH, DECISION to allow for more than $\frac{n}{8} - 1$ faulty processors.

## Exercise 7.3. (8) *Byzantine Agreement*

We consider the Byzantine Agreement algorithm 2.24 of the lecture notes.

We assume that the faulty processors know all random bits beforehand. Show that there are inputs for the reliable processors such that $\frac{n}{8} - 1$ faulty processors may prevent agreement.